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Mori-Tanaka approach extended to hygroelastic loading of fiber-reinforced composites

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Comparison with Eshelby-Kröner self-consistent model

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Abstract

The scale-transition model historically proposed by Mori and Tanaka in order to predict the average and local elastic behaviour of heterogeneous structures is for the first time extended to hygro-elastic load. Explicit constitutive laws satisfying to the fundamental assumptions of the model are given for the determination of the effective macroscopic Coefficients of Moisture Expansion (CME) in composite structures by considering a jump in moisture content between the fiber and the matrix. Explicit forms are also given for the calculation of local (fiber and matrix scale) internal stress states from the localization of the macroscopic hygro-mechanical states (ply scale).

Comparisons for several compositions of composite structures (volume fraction of the constituents, i.e. the epoxy matrix and the reinforcing fibers) are performed between the numerical predictions given by Mory-Tanaka model extended to hygro-elastic load and the recently proposed Eshelby-Kröner self-consistent hygro-elastic model. Discrepancies in the calculations appearing with an increase of the reinforcing fiber volume fraction are extensively discussed in order to conclude about the limitations of the micro-mechanical approach developed in the present work.

Key words: Mori-Tanaka model, Eshelby-Kröner self-consistent model, hygroelastic stresses, fiber-reinforced composites.

1. Introduction

For twenty years, composite laminates are considered as an interesting substitute to metallic or polymer materials, in the cases when high strength-to-weight ratio as well as corrosion and fatigue resistance are required in engineering applications. Since carbon/epoxy composites can absorb significant amount of water and exhibit heterogeneous Coefficients of Moisture Expansion (CME) (i.e. the CME of the epoxy matrix are strongly different from the CME of the carbon fibers), local stresses occur from a hygro-elastic loading of composite structures. Scale transition models providing formalism for the calculation of these local mechanical states are required in order to estimate a possible damage occurrence in such materials exposed to hygroscopic environments.

In previous works, Eshelby-Kröner self-consistent model [1-2] was extended to hygro-elastic loads [3]. Nevertheless, the programming of this model remains complex, due to the implicit determination of the effective macroscopic properties of the plies, even if closed-form

solutions could be expressed for the local mechanical states in very specific cases [4]. Now, in practice, one other scale transition model, also based on the mathematical work of Eshelby [1], but providing explicit forms for both the effective macroscopic properties of the plies and the local mechanical states is often preferred in order to treat the case of composite structures: the Mori and Tanaka approach [5]. This model, firstly introduced for pure elastic load has been extended to plastic and thermal load too: examples of numerical applications of this model to the case of composite structures can be found in [6-8] (elastic-plastic modelling of metal and polymer matrix composites) and [9] (determination of thermal stresses in metal matrix composites). Until now, the available scientific works based on Mori and Tanaka hypotheses do not provide the possibility to apply this model to treat the cases of composites submitted to hygroscopic loads.

First, a rigorous extension of Mori and Tanaka model to the case of hygro-elastic load will be described. The determination of both the effective macroscopic hygro-elastic properties (the elastic stiffness and the CME) and the local mechanical states will be detailed in that section. The next section shows comparisons of the numerical results obtained with either the Mori-Tanaka approach or the recently, extended to hygro-elastic loads Eshelby-Kröner model, as a function of the volume fraction of reinforcing fibers constituting the plies. The comparison of the predicted effective macroscopic properties and mechanical states (at both macroscopic and local scales) are discussed in the same section: in particular, the limit of validity of Mori and Tanaka approach to the cases of dilute solutions (small fiber volume fractions) underlined by Benveniste in pure elasticity [10] is confirmed for hygro-elastic loads.

2. Scale transition modeling for the hygro-elastic behaviour of composites

2.1 Introduction

Scale transition models are based on a multi-scale representation of materials. In the case of composite materials, for instance, a two-scale model is sufficient:

- The averaged behaviour of a ply, defines the macroscopic scale of the model. It is denoted by the superscript ^{I}.
- The properties and mechanical states of either the matrix or fibers are respectively indicated by the superscripts ^{m} and ^{f}. These constituents define the so-called "pseudo-macroscopic" scale of the material [11].

In general, scale transition models are assumed to satisfy the following relations on the stresses ($\boldsymbol{\sigma}$) and strains, ($\boldsymbol{\varepsilon}$) demonstrated by Hill [12]:

a) conditions over the averages of local stresses and strains,

$$\begin{aligned} \left\langle \boldsymbol{\sigma}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} &= \boldsymbol{\sigma}^{\mathbf{I}} \\ \left\langle \boldsymbol{\varepsilon}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} &= \boldsymbol{\varepsilon}^{\mathbf{I}} \end{aligned} \qquad (1)$$

In (1), the brackets <> stands for volume average operations. Introducing v^f and v^m as the volume fractions of fibers and matrix (resin) in the composite structure, it comes, as an example, for the stresses:

$$\left\langle \boldsymbol{\sigma}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} = \sum_{\boldsymbol{\alpha}} v^{\alpha} \boldsymbol{\sigma}^{\boldsymbol{\alpha}} = v^{m} \boldsymbol{\sigma}^{\mathbf{m}} + v^{f} \boldsymbol{\sigma}^{f} \qquad (2)$$

b) Hill's theorem:

$$\left\langle \boldsymbol{\sigma}^{\alpha} : \boldsymbol{\varepsilon}^{\alpha} \right\rangle_{\alpha=f,m} = \left\langle \boldsymbol{\sigma}^{\alpha} \right\rangle_{\alpha=f,m} : \left\langle \boldsymbol{\varepsilon}^{\alpha} \right\rangle_{\alpha=f,m} = \boldsymbol{\sigma}^{\mathbf{I}} : \boldsymbol{\varepsilon}^{\mathbf{I}} \tag{3}$$

Let us introduce **B** as the elastic stress concentration tensor linking the local stresses to the macroscopic ones in pure elasticity:

$$\boldsymbol{\sigma}^{\alpha} = \mathbf{B}^{\alpha} : \boldsymbol{\sigma}^{\mathbf{I}} \tag{4}$$

Taking into account (4) in (3) enables to demonstrate the following useful relation, after elementary tensorial calculation:

$$\boldsymbol{\sigma}^{\mathbf{I}} : \boldsymbol{\varepsilon}^{\mathbf{I}} = \left\langle \boldsymbol{\sigma}^{\alpha} : \boldsymbol{\varepsilon}^{\alpha} \right\rangle_{\alpha=f,m} = \left\langle \mathbf{B}^{\alpha} : \boldsymbol{\sigma}^{\mathbf{I}} : \boldsymbol{\varepsilon}^{\alpha} \right\rangle_{\alpha=f,m} = \left\langle \boldsymbol{\sigma}^{\mathbf{I}} : \mathbf{B}^{\alpha^{\mathbf{T}}} : \boldsymbol{\varepsilon}^{\alpha} \right\rangle_{\alpha=f,m} = \boldsymbol{\sigma}^{\mathbf{I}} : \left\langle \mathbf{B}^{\alpha^{\mathbf{T}}} : \boldsymbol{\varepsilon}^{\alpha} \right\rangle_{\alpha=f,m} \tag{5}$$

$\boldsymbol{\sigma}^{\mathbf{I}}$ appearing in both the right and left members of equation (5), the macroscopic strain generally satisfies:

$$\boldsymbol{\varepsilon}^{\mathbf{I}} = \left\langle \mathbf{B}^{\alpha^{\mathbf{T}}} : \boldsymbol{\varepsilon}^{\alpha} \right\rangle_{\alpha=f,m} \tag{6}$$

Now, under hygro-elastic loads, macroscopic and local strains are separated in two parts: the elastic strains and the hygroscopic strains, respectively denoted in the following by the subscripts $_{el}$ and $_{hy}$:

$$\begin{aligned} \boldsymbol{\varepsilon}^{\mathbf{I}} &= \boldsymbol{\varepsilon}^{\mathbf{I}}_{\mathbf{el}} + \boldsymbol{\varepsilon}^{\mathbf{I}}_{\mathbf{hy}} \\ \boldsymbol{\varepsilon}^{\boldsymbol{\alpha}} &= \boldsymbol{\varepsilon}^{\boldsymbol{\alpha}}_{\mathbf{el}} + \boldsymbol{\varepsilon}^{\boldsymbol{\alpha}}_{\mathbf{hy}} \end{aligned} \qquad (7)$$

The elastic strains are linearly related to the stress states through the stiffness tensor **L**, whereas the hygroscopic dilatation generated by a moisture content increment ΔC is treated as a transformation strain, using an approach similar to the one developed in order to take into account thermal strains (see for instance [13]) except that (i) the Coefficients of Moisture Expansion (CME) $\boldsymbol{\beta}$ replace the Coefficients of Thermal Expansion and (ii) the moisture contents ΔC^I, ΔC^m and ΔC^f are strongly different. It finally comes:

$$\begin{aligned} \boldsymbol{\varepsilon}^{\mathbf{I}} &= \boldsymbol{\varepsilon}^{\mathbf{I}}_{\mathbf{el}} + \boldsymbol{\varepsilon}^{\mathbf{I}}_{\mathbf{hy}} = \mathbf{L}^{\mathbf{I}^{-1}} : \boldsymbol{\sigma}^{\mathbf{I}} + \boldsymbol{\beta}^{\mathbf{I}} \Delta C^{I} \\ \boldsymbol{\varepsilon}^{\boldsymbol{\alpha}} &= \boldsymbol{\varepsilon}^{\boldsymbol{\alpha}}_{\mathbf{el}} + \boldsymbol{\varepsilon}^{\boldsymbol{\alpha}}_{\mathbf{hy}} = \mathbf{L}^{\boldsymbol{\alpha}^{-1}} : \boldsymbol{\sigma}^{\boldsymbol{\alpha}} + \boldsymbol{\beta}^{\boldsymbol{\alpha}} \Delta C^{\alpha} \end{aligned} \qquad (8)$$

2.2 General forms for the macroscopic hygro-elastic properties of heterogeneous composite structures

Within scale transition modeling, the local properties of the α–indiced constituents are usually considered to be known (i.e. $\mathbf{L}^{\boldsymbol{\alpha}}$ and $\boldsymbol{\beta}^{\boldsymbol{\alpha}}$), whereas the effective macroscopic properties of the composite structure are a priori unknown and results from computations. Relations (6-7) which should be satisfied for any load, provides two general relations enabling the determination of the missing macroscopic stiffness and CME:

(i) In order to determine the effective macroscopic elastic stiffness of an heterogeneous structure through scale transition models, the case of a purely elastic load is generally considered [13]. As a consequence, the moisture contents are null: $\Delta C^I = \Delta C^{\alpha} = 0$. As a result, the hygroscopic part of the

strains vanishes and the pseudo-macroscopic strains writes: $\boldsymbol{\varepsilon}^{\boldsymbol{\alpha}} = \boldsymbol{\varepsilon}_{\mathbf{el}}^{\boldsymbol{\alpha}} = \mathbf{L}^{\boldsymbol{\alpha}^{-1}} : \boldsymbol{\sigma}^{\boldsymbol{\alpha}} = \mathbf{L}^{\boldsymbol{\alpha}^{-1}} : \mathbf{B}^{\boldsymbol{\alpha}} : \boldsymbol{\sigma}^{\mathbf{I}}$. Introducing these conditions in (8) yields the following form of equation (6):

$$\begin{aligned} \boldsymbol{\varepsilon}^{\mathbf{I}} &= \boldsymbol{\varepsilon}_{\mathbf{el}}^{\mathbf{I}} = \mathbf{L}^{\mathbf{I}^{-1}} : \boldsymbol{\sigma}^{\mathbf{I}} \\ &= \left\langle \mathbf{B}^{\boldsymbol{\alpha}^{\mathbf{T}}} : \boldsymbol{\varepsilon}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} = \left\langle \mathbf{B}^{\boldsymbol{\alpha}^{\mathbf{T}}} : \boldsymbol{\varepsilon}_{\mathbf{el}}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} = \left\langle \mathbf{B}^{\boldsymbol{\alpha}^{\mathbf{T}}} : \mathbf{L}^{\boldsymbol{\alpha}^{-1}} : \boldsymbol{\sigma}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} = \left\langle \boldsymbol{\sigma}^{\boldsymbol{\alpha}} : \mathbf{L}^{\boldsymbol{\alpha}^{-1}} : \mathbf{B}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} \end{aligned} \tag{9}$$

Besides, Hill's theorem (3) can be written as follows:

$$\left\langle \boldsymbol{\sigma}^{\boldsymbol{\alpha}} : \boldsymbol{\varepsilon}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} = \left\langle \boldsymbol{\sigma}^{\boldsymbol{\alpha}} : \mathbf{L}^{\boldsymbol{\alpha}^{-1}} : \mathbf{B}^{\boldsymbol{\alpha}} : \boldsymbol{\sigma}^{\mathbf{I}} \right\rangle_{\alpha=f,m} = \left\langle \boldsymbol{\sigma}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} : \left\langle \boldsymbol{\varepsilon}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} = \left\langle \boldsymbol{\sigma}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} : \left\langle \mathbf{L}^{\boldsymbol{\alpha}^{-1}} : \mathbf{B}^{\boldsymbol{\alpha}} : \boldsymbol{\sigma}^{\mathbf{I}} \right\rangle_{\alpha=f,m} \tag{10}$$

Since $\boldsymbol{\sigma}^{\mathbf{I}}$ is a constant, the following identity is obtained:

$$\left\langle \boldsymbol{\sigma}^{\boldsymbol{\alpha}} : \mathbf{L}^{\boldsymbol{\alpha}^{-1}} : \mathbf{B}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} = \left\langle \boldsymbol{\sigma}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} : \left\langle \mathbf{L}^{\boldsymbol{\alpha}^{-1}} : \mathbf{B}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} = \boldsymbol{\sigma}^{\mathbf{I}} : \left\langle \mathbf{L}^{\boldsymbol{\alpha}^{-1}} : \mathbf{B}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} \tag{11}$$

Inserting (11) in (9) yields: $\mathbf{L}^{\mathbf{I}^{-1}} : \boldsymbol{\sigma}^{\mathbf{I}} = \boldsymbol{\sigma}^{\mathbf{I}} : \left\langle \mathbf{L}^{\boldsymbol{\alpha}^{-1}} : \mathbf{B}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m}$. Consequently, by identification, the elastic stiffness satisfies the relation:

$$\mathbf{L}^{\mathbf{I}} = \left\langle \mathbf{L}^{\boldsymbol{\alpha}^{-1}} : \mathbf{B}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m}^{-1} \tag{12}$$

(ii) In order to determine the effective macroscopic Coefficients of Thermal Expansion (CTE) of an heterogeneous structure through scale transition

models, the case of a purely thermal load is generally considered [13]. The problem of determining CME instead of CTE is rather similar, β and ΔC replacing the CTE and temperature increment ΔT, respectively. Thus, by analogy with the thermo-elastic case, finding out the effective macroscopic CME of an heterogeneous structure using scale transition model implies to consider the case that a purely hygroscopic load is applied to the material. Consequently, the macroscopic stress is null: $\boldsymbol{\sigma}^{\mathbf{I}}=\mathbf{0}$. As a result, the elastic part of the macroscopic strains vanishes: $\boldsymbol{\varepsilon}_{\mathbf{el}}^{\mathbf{I}}=\mathbf{0}$, and $\boldsymbol{\varepsilon}^{\mathbf{I}}=\boldsymbol{\varepsilon}_{\mathbf{hy}}^{\mathbf{I}}=\boldsymbol{\beta}^{\mathbf{I}}\Delta C^{I}$. Besides, since $\boldsymbol{\varepsilon}_{\mathbf{el}}^{\mathbf{I}}=\mathbf{0}$, the following average has also to be null in equation (6): $\left\langle \mathbf{B}^{\boldsymbol{\alpha}^{\mathbf{T}}} : \boldsymbol{\varepsilon}_{\mathbf{el}}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m}=\mathbf{0}$. Introducing these conditions in (8) yields the following form of equation (6):

$$\begin{aligned}\boldsymbol{\varepsilon}^{\mathbf{I}} &= \boldsymbol{\beta}^{\mathbf{I}}\Delta C^{I} \\ &= \left\langle \mathbf{B}^{\boldsymbol{\alpha}^{\mathbf{T}}} : \boldsymbol{\varepsilon}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} = \left\langle \mathbf{B}^{\boldsymbol{\alpha}^{\mathbf{T}}} : \boldsymbol{\varepsilon}_{\mathbf{hy}}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} = \left\langle \mathbf{B}^{\boldsymbol{\alpha}^{\mathbf{T}}} : \boldsymbol{\beta}^{\boldsymbol{\alpha}}\Delta C^{\alpha} \right\rangle_{\alpha=f,m}\end{aligned} \tag{13}$$

From equation (13), the identification of the general form providing the effective macroscopic CME of an heterogeneous structure is obvious:

$$\boldsymbol{\beta}^{\mathbf{I}}=\frac{1}{\Delta C^{I}}\left\langle \mathbf{B}^{\boldsymbol{\alpha}^{\mathbf{T}}} : \boldsymbol{\beta}^{\boldsymbol{\alpha}}\Delta C^{\alpha} \right\rangle_{\alpha=f,m} \tag{14}$$

The forms (12) and (14) presented in this section are general: they do not depend on the choice of the scale transition model nor on the hygro-elastic load effectively applied on the structure (in the case that the material properties are assumed to be independent from the

hygro-mechanical state). Actually, the constitutive hypotheses of the considered model for the purpose of performing calculations will provide specific (i.e. model dependent) relations for the stress localization tensor $\mathbf{B}^{\alpha}$ involved in (12, 14). The following subsection gives an illustration in the case that Mori and Tanaka approximation is taken into account.

2.3 Mori and Tanaka model extended to hygro-elastic loading of composite structures: macroscopic effective properties and local stress states

2.3.1 Introduction to Mori and Tanaka model: constitutive assumptions

For simplicity, the case of two-phases composites only will be considered here. Nevertheless, the approach detailed in the present subsection could be generalized to any number of phases. Similarly to the case of Kröner and Eshelby self-consistent model, Mori and Tanaka theory is closely attached to the equivalent inclusion idea of Eshelby. In this framework, the material is divided in a matrix phase (in the present case, the epoxy matrix) and an inclusion phase (the carbon fibers), the last one being constituted of ellipsoidal particles either aligned or randomly oriented. No particular assumption is made on the geometry of the matrix. The two phases are possibly elastically and/or hygroscopically anisotropic.

In this theory, contrary to the case of Eshelby-Kröner self-consistent model, the inclusions are not considered to be embedded directly in the effective model having the behaviour of the composite structure: in Mori and Tanaka approximation, the particles (fibers) are embedded in the matrix phase, itself being loaded at the infinite by the hygro-mechanical conditions applied on the composite structure. In consequence, the inclusion phase does not experience any interaction with the macroscopic scale, but with the matrix only. In consequence, Mori and Tanaka model corresponds to the direct extension of Eshelby's single inclusion model [2]

to the case that the volume fraction of inclusions does not remain infinitesimal anymore. Thus, the scale transition relations holding for Eshelby's dilute model are assumed to still be valid within Mori and Tanaka procedure. In particular, the stress localization tensor $\mathbf{B}^{\alpha}$ satisfies (an extensive demonstration, yielding an equivalent form, is available in [10]):

$$\mathbf{B}^{\alpha} = \mathbf{L}^{\alpha} : \mathbf{T}^{\alpha} : \left\langle \mathbf{L}^{\alpha} : \mathbf{T}^{\alpha} \right\rangle^{-1} \tag{15}$$

where the elastic strain localization tensor, usually denoted by $\mathbf{T}^{\alpha}$ within Mori and Tanaka approach, is written as follows:

$$\mathbf{T}^{\alpha} = \left[\mathbf{I} + \mathbf{E}^{\alpha} : \left(\mathbf{L}^{\alpha} - \mathbf{L}^{\mathbf{m}} \right) \right]^{-1} \tag{16}$$

In (16), $\mathbf{I}$ stands for the fourth order identity tensor. Hill's tensor $\mathbf{E}^{\alpha}$ expresses the dependence of the localization tensor on the morphology assumed for the matrix and its reinforcements [14]. It can be expressed as a function of Eshelby's tensor $\mathbf{S}^{\alpha}_{\mathbf{esh}}$, through $\mathbf{E}^{\alpha} = \mathbf{S}^{\alpha}_{\mathbf{esh}} : \mathbf{L}^{\mathbf{m}^{-1}}$. In practice, the calculation of Hill's tensor for the inclusion phase $\mathbf{E}^{\mathbf{f}}$ only is necessary, since obvious simplifications of (16), leading to $\mathbf{T}^{\mathbf{m}} = \mathbf{I}$, occur in the case that the matrix is considered.

2.3.2 Macroscopic effective properties in Mori and Tanaka framework

In order to find the effective macroscopic properties of the plies constituting the composite structure, the specific stresses and strains localization tensors (15-16) have to be inserted in the general forms (12, 14) previously obtained for both the macroscopic stiffness and CME.

$$\mathbf{L}^{\mathbf{I}} = \left\langle \mathbf{L}^{\boldsymbol{\alpha}\,-1} : \mathbf{B}^{\boldsymbol{\alpha}} \right\rangle^{-1}_{\alpha=f,m} = \left\langle \mathbf{T}^{\boldsymbol{\alpha}} : \left\langle \mathbf{L}^{\boldsymbol{\alpha}} : \mathbf{T}^{\boldsymbol{\alpha}} \right\rangle^{-1}_{\alpha=f,m} \right\rangle^{-1}_{\alpha=f,m} \tag{17}$$

$$\boldsymbol{\beta}^{\mathbf{I}} = \frac{1}{\Delta C^{I}} \left\langle \mathbf{B}^{\boldsymbol{\alpha}\mathbf{T}} : \boldsymbol{\beta}^{\boldsymbol{\alpha}} \Delta C^{\alpha} \right\rangle_{\alpha=f,m} = \frac{1}{\Delta C^{I}} \left\langle \left(\mathbf{L}^{\boldsymbol{\alpha}} : \mathbf{T}^{\boldsymbol{\alpha}} : \left\langle \mathbf{L}^{\boldsymbol{\alpha}} : \mathbf{T}^{\boldsymbol{\alpha}} \right\rangle^{-1} \right)^{\mathbf{T}} : \boldsymbol{\beta}^{\boldsymbol{\alpha}} \Delta C^{\alpha} \right\rangle_{\alpha=f,m} \tag{18}$$

Since the moisture concentration in the fiber is usually null, the introduction of this additional assumption (i.e. $\Delta C^{f} = 0$) provides the following simplifications of (18):

$$\boldsymbol{\beta}^{\mathbf{I}} = v^{m} \frac{\Delta C^{m}}{\Delta C^{I}} \mathbf{L}^{\mathbf{m}} : \left\langle \mathbf{L}^{\boldsymbol{\alpha}} : \mathbf{T}^{\boldsymbol{\alpha}} \right\rangle^{-1}_{\alpha=f,m} : \boldsymbol{\beta}^{\mathbf{m}} \tag{19}$$

Contrary to the case, previously detailed in [3-4], that Eshelby-Kröner self-consistent model is used to estimate the macroscopic effective hygro-mechanical properties of composite structures, both the stiffness and CME are explicit according to Mori and Tanaka model.

2.3.3 Local hygro-elastic stresses and strains according to Mori and Tanaka model

In this subsection, the macroscopic effective properties are assumed to be known from numerical applications of (18-19). Thus (8) can be applied in order to find either the macroscopic stress or strain state, from the considered hygro-elastic load.

The main question to solve is to express the scale transition relation linking the macroscopic hygro-mechanical states to their corresponding local counterparts. For reasons of analogies existing between these two kinds of loads, the scale transition relations that have previously been established for both pure elastic and pure thermal loads within Mori and Tanaka

framework will be derived to enable the treatment of the hygro-elastic load. In the case that a thermo-elastic problem is considered (i.e. a macroscopic load $\{\boldsymbol{\sigma}^{\mathbf{I}}, \Delta T^{I}=\Delta T^{m}=\Delta T^{f}\}$ is applied), the local mechanical states are identical to the sum of the separated contributions to: i) the pure elastic stress and ii) the pure thermal load ΔT^{I}:

i) According to the definition of the elastic stress localization tensor, the elastic part of the local stresses satisfies relation (4).

ii) For a pure thermal load, the reinforcing phase is assumed to be subjected, at the infinite, to the stress state experienced by the embedding matrix of stiffness $\mathbf{L}^{\mathbf{m}}$. The reaction tensor, modulating the stress reaction induced by the strains deviation between the fiber inclusion and the matrix is equal to $\mathbf{S}_{\mathbf{esh}}^{\mathbf{f}\,-1} - \mathbf{I}$. The strains deviation in itself corresponds to the sum of both the thermal expansion $\Delta\boldsymbol{\varepsilon}_{\mathbf{T}}$ and elastic stiffnesses $\Delta\boldsymbol{\varepsilon}_{\mathbf{S}}$ mismatches, which respectively reads:

$$\Delta\boldsymbol{\varepsilon}_{\mathbf{T}} = \mathbf{M}^{\mathbf{f}}\Delta T^{f} - \mathbf{M}^{\mathbf{m}}\Delta T^{m} = \left(\mathbf{M}^{\mathbf{f}} - \mathbf{M}^{\mathbf{m}}\right)\Delta T^{I} \tag{20}$$

$$\Delta\boldsymbol{\varepsilon}_{\mathbf{S}} = \left(\mathbf{L}^{\mathbf{f}\,-1} - \mathbf{L}^{\mathbf{m}\,-1}\right) : \boldsymbol{\sigma}^{\mathbf{f}} \tag{21}$$

where **M** stands for the Coefficients of Thermal Expansion.

Consequently, according to the classical formalism of Eshelby [2], the stress in the inclusion (fiber) submitted to a thermal treatment satisfies the following general form:

$$\begin{aligned}\boldsymbol{\sigma}^{\mathbf{f}} - \boldsymbol{\sigma}^{\mathbf{m}} &= -\mathbf{L}^{\mathbf{m}} : \left(\mathbf{S}_{\mathbf{esh}}^{\mathbf{f}\,-1} - \mathbf{I}\right) : \left[\Delta\boldsymbol{\varepsilon}_{\mathbf{T}} + \Delta\boldsymbol{\varepsilon}_{\mathbf{S}}\right] \\ &= -\mathbf{L}^{\mathbf{m}} : \left(\mathbf{S}_{\mathbf{esh}}^{\mathbf{f}\,-1} - \mathbf{I}\right) : \left[\left(\mathbf{M}^{\mathbf{f}} - \mathbf{M}^{\mathbf{m}}\right)\Delta T^{I} + \left(\mathbf{L}^{\mathbf{f}\,-1} - \mathbf{L}^{\mathbf{m}\,-1}\right) : \boldsymbol{\sigma}^{\mathbf{f}}\right]\end{aligned} \tag{22}$$

By analogy with Mori-Tanaka thermo-elastic model, the expression enabling to determine the local stresses and strains experienced by the matrix and fiber when a macroscopic load $\{\boldsymbol{\sigma}^{\mathbf{I}}, \Delta C^{I}\}$ is applied on a given ply can be provided by the sum of the separated response of the elementary constituents of the ply to: j) the pure elastic load $\boldsymbol{\sigma}^{\mathbf{I}}$ and jj) the pure hygroscopic load ΔC^{I}:

j) Identically to i) the elastic part of the local stresses has to satisfy relation (4).

jj) The treatment of a pure hygroscopic load, is analogous to the case of a pure thermal load ii). The only modification concerns the strains deviations. In the present case, the thermal mismatch as to be replaced by the corresponding hygroscopic mismatch $\Delta\boldsymbol{\varepsilon}_{\mathbf{H}}$. It means that equation (20) transforms as follows (whereas the contribution due to elastic stiffnesses mismatch (21) remains unchanged):

$$\Delta\boldsymbol{\varepsilon}_{\mathbf{H}} = \boldsymbol{\beta}^{\mathbf{f}}\Delta C^{f} - \boldsymbol{\beta}^{\mathbf{m}}\Delta C^{m} \tag{23}$$

As a consequence, the average stress in the inclusion (fiber) of a composite structure submitted to a pure hygroscopic load satisfies the following general form deduced from (22), after replacement of term (20) by (23):

$$\boldsymbol{\sigma}^{\mathbf{f}} - \boldsymbol{\sigma}^{\mathbf{m}} = -\mathbf{L}^{\mathbf{m}} : \left(\mathbf{S}_{\mathbf{esh}}^{\mathbf{f}\,-1} - \mathbf{I}\right) : \left[\boldsymbol{\beta}^{\mathbf{f}}\Delta C^{f} - \boldsymbol{\beta}^{\mathbf{m}}\Delta C^{m} + \left(\mathbf{L}^{\mathbf{f}\,-1} - \mathbf{L}^{\mathbf{m}\,-1}\right) : \boldsymbol{\sigma}^{\mathbf{f}}\right] \tag{24}$$

Besides, for a purely hygroscopic load, $\left\langle \boldsymbol{\sigma}^{\boldsymbol{\alpha}} \right\rangle_{\alpha=f,m} = \mathbf{0}$, thus, $\boldsymbol{\sigma}^{\mathbf{f}} = -\dfrac{V^{m}}{V^{f}}\boldsymbol{\sigma}^{\mathbf{m}}$. Moreover, $\Delta C^{f} = 0$. After simplifications of (24), the local hygroscopic stresses are obtained:

$$\boldsymbol{\sigma}^{\mathbf{m}} = \mathrm{v}^{\mathrm{m}}\mathbf{B}^{\mathbf{m}} : \left[\mathbf{I} - \mathbf{L}^{\mathbf{m}} : \left(\mathbf{S}_{\mathbf{esh}}^{\mathbf{f}-1} - \mathbf{I}\right) : \left(\mathbf{L}^{\mathbf{f}} - \mathbf{L}^{\mathbf{m}}\right)\right]^{-1} : \mathbf{L}^{\mathbf{m}} : \left(\mathbf{S}_{\mathbf{esh}}^{\mathbf{f}-1} - \mathbf{I}\right) : \boldsymbol{\beta}^{\mathbf{m}}\Delta\mathrm{C}^{\mathrm{m}} \qquad (25)$$

$$\boldsymbol{\sigma}^{\mathbf{f}} = \left(\mathrm{v}^{\mathrm{f}}\mathbf{B}^{\mathbf{f}} - \mathbf{I}\right) : \left[\mathbf{I} - \mathbf{L}^{\mathbf{m}} : \left(\mathbf{S}_{\mathbf{esh}}^{\mathbf{f}-1} - \mathbf{I}\right) : \left(\mathbf{L}^{\mathbf{f}} - \mathbf{L}^{\mathbf{m}}\right)\right]^{-1} : \mathbf{L}^{\mathbf{m}} : \left(\mathbf{S}_{\mathbf{esh}}^{\mathbf{f}-1} - \mathbf{I}\right) : \boldsymbol{\beta}^{\mathbf{m}}\Delta\mathrm{C}^{\mathrm{m}} \qquad (26)$$

The hygroscopic stresses are obviously self-equilibrated (this can be easily verified, since the elastic stress concentration tensor has the following main property: $\mathrm{v}^{\mathrm{f}}\mathbf{B}^{\mathbf{f}} + \mathrm{v}^{\mathrm{m}}\mathbf{B}^{\mathbf{m}} = \mathbf{I}$) and are directly proportional to the moisture content of the matrix.

Finally, the hygro-elastic pseudo-macroscopic stresses in the Mori and Tanaka approach are, as previously explained, expressed by the sum of the elastic (4) and hygroscopic local (25-26) stresses:

$$\boldsymbol{\sigma}^{\mathbf{m}} = \mathbf{B}^{\mathbf{m}} : \boldsymbol{\sigma}^{\mathbf{I}} + \mathrm{v}^{\mathrm{f}}\mathbf{B}^{\mathbf{m}} : \left[\mathbf{I} - \mathbf{L}^{\mathbf{m}} : \left(\mathbf{S}_{\mathbf{esh}}^{\mathbf{f}-1} - \mathbf{I}\right) : \left(\mathbf{L}^{\mathbf{f}} - \mathbf{L}^{\mathbf{m}}\right)\right]^{-1} : \mathbf{L}^{\mathbf{m}} : \left(\mathbf{S}_{\mathbf{esh}}^{\mathbf{f}-1} - \mathbf{I}\right) : \boldsymbol{\beta}^{\mathbf{m}}\Delta\mathrm{C}^{\mathrm{m}} \qquad (27)$$

$$\boldsymbol{\sigma}^{\mathbf{f}} = \mathbf{B}^{\mathbf{f}} : \boldsymbol{\sigma}^{\mathbf{I}} + \left(\mathrm{v}^{\mathrm{f}}\mathbf{B}^{\mathbf{f}} - \mathbf{I}\right) : \left[\mathbf{I} - \mathbf{L}^{\mathbf{m}} : \left(\mathbf{S}_{\mathbf{esh}}^{\mathbf{f}-1} - \mathbf{I}\right) : \left(\mathbf{L}^{\mathbf{f}} - \mathbf{L}^{\mathbf{m}}\right)\right]^{-1} : \mathbf{L}^{\mathbf{m}} : \left(\mathbf{S}_{\mathbf{esh}}^{\mathbf{f}-1} - \mathbf{I}\right) : \boldsymbol{\beta}^{\mathbf{m}}\Delta\mathrm{C}^{\mathrm{m}} \qquad (28)$$

The local strains are evaluated from (27-28) through the constitutive relation (8).

3 Numerical comparisons between Mori-Tanaka and Eshelby-Kröner self-consistent models extended to hygro-elastic loads

The present section aims to estimate the reliability of Mori and Tanaka model for the accurate prediction of both the overall effective behaviour of composite structures and the calculation

of local stress states in its elementary constituents. As explained in the subsection 2.3.1, Mori-Tanaka approach assumes that the formalism demonstrated by Eshelby for the rigorous treatment of a single inclusion, the volume fraction of which tends towards zero, remains true for any volume fraction of inclusions. In practice, Mori-Tanaka approach is built on the rough hypothesis, that the reinforcement are embedded in a matrix, which thickness is relatively large, in comparison with the thickness of the reinforcement itself, this condition involving that the embedded fibers can be considered not to directly interact with the effective medium (macroscopic scale), but with the epoxy matrix only. It was demonstrated by Benveniste in [10], for the pure elastic case, that this hypothesis was not always true, for instance, in polycrystals containing cracks. Now, in the case that carbon epoxy composites are considered, Mori-Tanaka fundamental hypothesis can be highly expected not to be fulfilled, since the thicknesses of the fibers and the surrounding matrix often reaches the same order of magnitude: thus, it is not rigorous to assume that the reinforcing inclusions do interact with the embedding matrix only and not at all with the macroscopic effective medium (ply). Since Mori and Tanaka procedure is, for the first time, extended to hygro-elastic load of composite structures in the present work, it is necessary to estimate the domain of validity of this model in terms of volume fraction of the inclusion phase (precisely, the carbon fibers).

In order to reach this goal the numerical predictions obtained through Mori and Tanaka approximation will be compared to the more rigorous Eshelby-Kröner self-consistent approach previously presented in [3-4], which properly extends Eshelby's fundamental work to the case when the volume fraction of inclusions is not infinitesimal anymore.

3.1 Comparison between macroscopic effective hygro-elastic properties of the plies

Both Mori and Tanaka approach and Eshelby-Kröner self-consistent hygro-elastic models [3-4] have been used in order to determine both the macroscopic elastic stiffness and coefficients of moisture expansion of carbon/epoxy composites T300/5208 as a function of T300 carbon fiber volume fraction, starting from v^f=0% up to v^f=60%. The behaviour of each composite is governed by its constituents, i.e. the properties of the fibers, the surrounding matrix and the assumed morphology for the inclusions. Calculations were performed taking into account the local properties listed in Table 1 and Table 2, for elastic constants and CME, respectively. In order to take into account the proper fiber microstructure of the material, the following values were taken for the length of the semi-axis of the inclusions: $a_2=a_3=1$, and $a_1\rightarrow\infty$, where the subscripts 1, 2, 3 respectively stand for the longitudinal, transverse and normal directions of the fibers. Close form solutions for Hill's tensor corresponding to this specific morphology was provided in reference [4] for Eshelby-Kröner model. In the case that Mori and Tanaka procedure is considered, Hill's tensor is calculated for an inclusion embedded in the epoxy matrix instead of the effective macroscopic medium, as it is the case within Eshelby-Kröner approach. As a consequence, any L_{ij}^I involved in the analytical form of Hill's tensor established for infinite fibers in Eshelby-Kröner model (equation (8) of reference [4]) has to be replaced by the corresponding L_{ij}^m in order to apply Mori and Tanaka model. In order to estimate the macroscopic CME, the ratio between the moisture content of the epoxy matrix and the one of the ply, i.e. $\frac{\Delta C^m}{\Delta C^I}$ is required. When the equilibrium state is reached, the maximum moisture content of the neat resin ΔC^m may actually be estimated from the maximum moisture content of the composite ΔC^I. By assuming that the fibers do not absorb any moisture, ΔC^I and ΔC^m are related by the expression given by Loos and Springer [17]:

$$\Delta C^I = \Delta C^m W^m \tag{29}$$

where W^m is the weight fraction (percent) of the resin in the composite.

The preceding equation develops as follows:

$$\begin{cases} \Delta C^m = \dfrac{c^m}{\rho^m} \\ \Delta C^I = \dfrac{c^I}{\rho^I} \\ c^I = (1-v^f)c^m \end{cases} \Rightarrow \frac{\Delta C^m}{\Delta C^I} = \frac{c^m \rho^I}{c^I \rho^m} = \frac{c^m \rho^I}{(1-v^f)c^m \rho^m} = \frac{\rho^I}{(1-v^f)\rho^m} \tag{30}$$

c and ρ are respectively the moisture concentration and the mass density of the dry material.The parameters and properties considered during the computations are: ρ^m=1200 kg/m^3, ρ^f=1867 kg/m^3 and c^I=1.5 %. Besides, the macroscopic density is obviously given by:

$$\rho^I = v^m \rho^m + v^f \rho^f \tag{31}$$

The numerical results obtained for both the longitudinal and transverse macroscopic Young's moduli and CME have been drawn on figure 1. Graphics displayed on figure 1 show the following remarkable facts:

i) Eshelby-Kröner and Mori-Tanaka hygro-elastic models do not predict identical macroscopic stiffnesses and CME for composite structures (in particular, the transverse Young's modulus and CME exhibit significant deviations from one model to the other).

ii) The discrepancies existing between the calculated macroscopic hygro-mechanical behaviour, according to Eshelby-Kröner model or Mori-Tanaka

approximation, depend on the fiber volume fraction: they are obviously null for a material containing 0% of carbon fibers, and increase with the volume fraction of carbon fibers.

iii) Nevertheless, these discrepancies of the hygro-mechanical properties are not critical: the evolution of some constants, like the longitudinal Young modulus E_1 and CME β_1 as a function of the volume fraction of carbon fiber is even almost identical. That is, the effective axial modulus follows, in first approximation, a rule of mixtures (its evolution as a function of fiber volume fraction is more or less linear). As a result, since they yield very similar numerical results, there is no fundamental reason (except the fact that Mori-Tanaka procedure constitutes only an approximation of Eshelby-Kröner rigorous mathematical treatment of the inclusion problem) to prefer Eshelby-Kröner model to Mori-Tanaka approach in the case that the effective macroscopic hygro-elastic properties of composite structures have to be estimated from the knowledge of the behaviour of their elementary constituents.

3.2 Comparison between the local mechanical states

In order to check the agreement between the local mechanical states predicted by Mori-Tanaka and Eshelby-Kröner models in both the epoxy resin and the carbon fiber of the plies of composite structures submitted to hygroscopic load, further computations were done.

Thin laminated composite pipes, with thickness 4 mm, initially dry, then exposed to an ambient fluid, made up of T300/5208 carbon-epoxy plies are considered for the determination of the macroscopic stresses. The closed-form formalism used in order to determine the

mechanical stresses and strains in each ply of the structure is described by Jacquemin and Vautrin [18]. The macroscopic effective hygro-mechanical properties necessary to perform these calculations correspond to the results of the computations presented in the previous subsection 3.1.

The macroscopic mechanical states were calculated in a $[55°/-55°]_S$ laminated cylinder, at the equilibrium of the moisture concentration, i.e. the time when the space dependent macroscopic moisture concentration becomes identical to the moisture concentration corresponding to the boundary conditions: $c^I(r) = 0.015$. Thereafter, the local mechanical states were determined, in the central ply of the laminate (in this ply, the macroscopic stresses are almost constant), using for instance equations (27-28) for Mori-Tanaka model. The equivalent equations satisfied in Eshelby-Kröner approach can be found in [3-4]. The evolution of the non-zero stress components at macroscopic (ply) and local (epoxy resin and carbon fibers) scales, as a function of the proportion of carbon fiber in the structure, is depicted on figure 2. The main observations are as follows:

i) The macroscopic stresses estimated using continuum mechanics formalism from the effective macroscopic properties determined according to either Eshelby-Kröner or Mori-Tanaka models are very close together. The discrepancies occurring in σ^I components increase proportionally to the fiber volume fraction. The explanation of this is directly related to the increasing discrepancies as a function of the fiber concentration between the macroscopic hygro-elastic properties previously observed in subsection 3.1. Consequently, the same conclusion as before holds: Mori-Tanaka approximation is in good agreement with Eshelby-Kröner model for calculating input values involved in macroscopic mechanical states determination in composite laminates submitted

to hygroscopic loads, even in the cases when a substantial amount of fibers is present in the considered material.

ii) At the contrary, the local stresses in both the fibers and the epoxy resin can, according to figure 2, strongly depend on the scale-transition model used to perform the calculations.

iii) As usual, the discrepancies between the two estimations of the local stresses increase with the fiber volume fraction.

iv) Besides, in the present case, the deviation existing between the local stresses predicted with Eshelby-Kröner or Mori-Tanaka approximation are not always close together. Significant deviation can even occur at weak carbon fibers contents: it is especially the case for the shear and normal stresses (see the evolutions obtained for the local stress in the reinforcing fibers). This result comes from the different scale transition relations assumed to hold within the two models.

v) Mori and Tanaka approximation generally underestimate the absolute value of the local stresses, in comparison with the corresponding predictions of Eshelby-Kröner model. In the case that the local stress states are calculated according to Mori-Tanaka model in order to estimate a possible damage occurrence of the constituents of the plies, this could lead to a dangerous overestimation of their domain of resistance.

vi) In conclusion, Mori and Tanaka model extended to the hygro-elastic loading of composite structures should only be considered as a rough approximation of the more rigorous Eshelby-Kröner model for the purpose of determining the local mechanical states. A satisfying agreement between the two models fix the domain of reliability of Mori-Tanaka hygro-elastic model to carbon fibers

volume fractions weaker than 10%. This result is compatible with the numerical computations performed in pure elasticity, in previous works [10].

4 Conclusions and perspectives

The present work details an extension to hygro-elastic loading of Mori-Tanaka procedure, which is often used in the field of the micro-mechanical modelling of composites structures, instead of the more rigorous, but also more complicated Eshelby-Kröner approach. The hygro-elastic version of Mori-Tanaka model enables the estimation of both the, possibly anisotropic, effective stiffness and coefficients of moisture expansion of the composite structure at macroscopic scale. The microstructure of the constituents, and, in particular, the morphology of the reinforcing fibers is properly taken into account in the model. Moreover, scale transition relations are provided for the estimation of the heterogeneous local mechanical states existing in the epoxy resin and the carbon fibers, of a given ply of the structure.

Applications of the new extension of Mori-Tanaka approach to the case of T300/N5208 $[55°/-55°]_S$ laminated cylinders exposed to an ambient fluid are provided. Comparisons with the numerical predictions of the recently proposed Eshelby-Kröner hygro-elastic model have shown that the two models are in good agreement in the case that the macroscopic behaviour and mechanical states of the composite structure only have to be determined. In the case that the local hygro-elastic stresses are considered, Mori and Tanaka model always underestimates the absolute value of the pseudo-macroscopic mechanical states. As a result, the domain of reliability of the model is limited to small volume fractions of reinforcing fibers: weaker than 10%. In the cases when the composite structures contains a volume fraction of fiber being

higher than 10%, Eshelby-Kröner model should be preferred to Mori-Tanaka estimates. This last result is compatible with previous works performed in pure elasticity.

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Figures captions

Figure 1. Evolution of longitudinal and transverse macroscopic elastic moduli and CME for T300/5208 composites. Comparison between Mori-Tanaka and Eshelby-Kröner models.

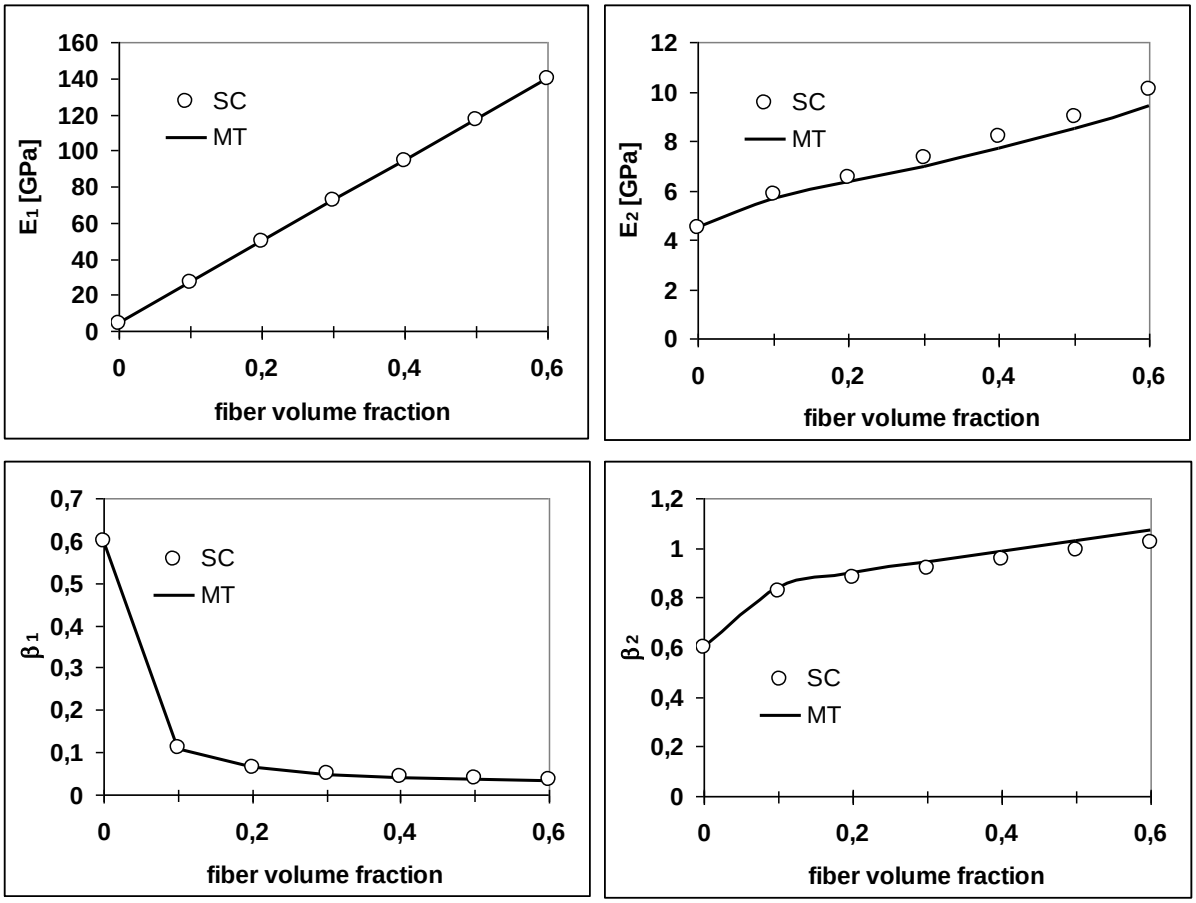
Figure 2. Macroscopic and local stresses in T300/5208 composites for the central ply of $[+55°/-55°]_S$ symmetric laminate, as a function of carbon fiber volume fraction. Comparison between Eshelby-Kröner and Mori-Tanaka predictions.

Tables captions

Table 1. Mechanical properties of the constituents of the studied composites.

Table 2. CME for the components of T300/5208 composite.

Figures



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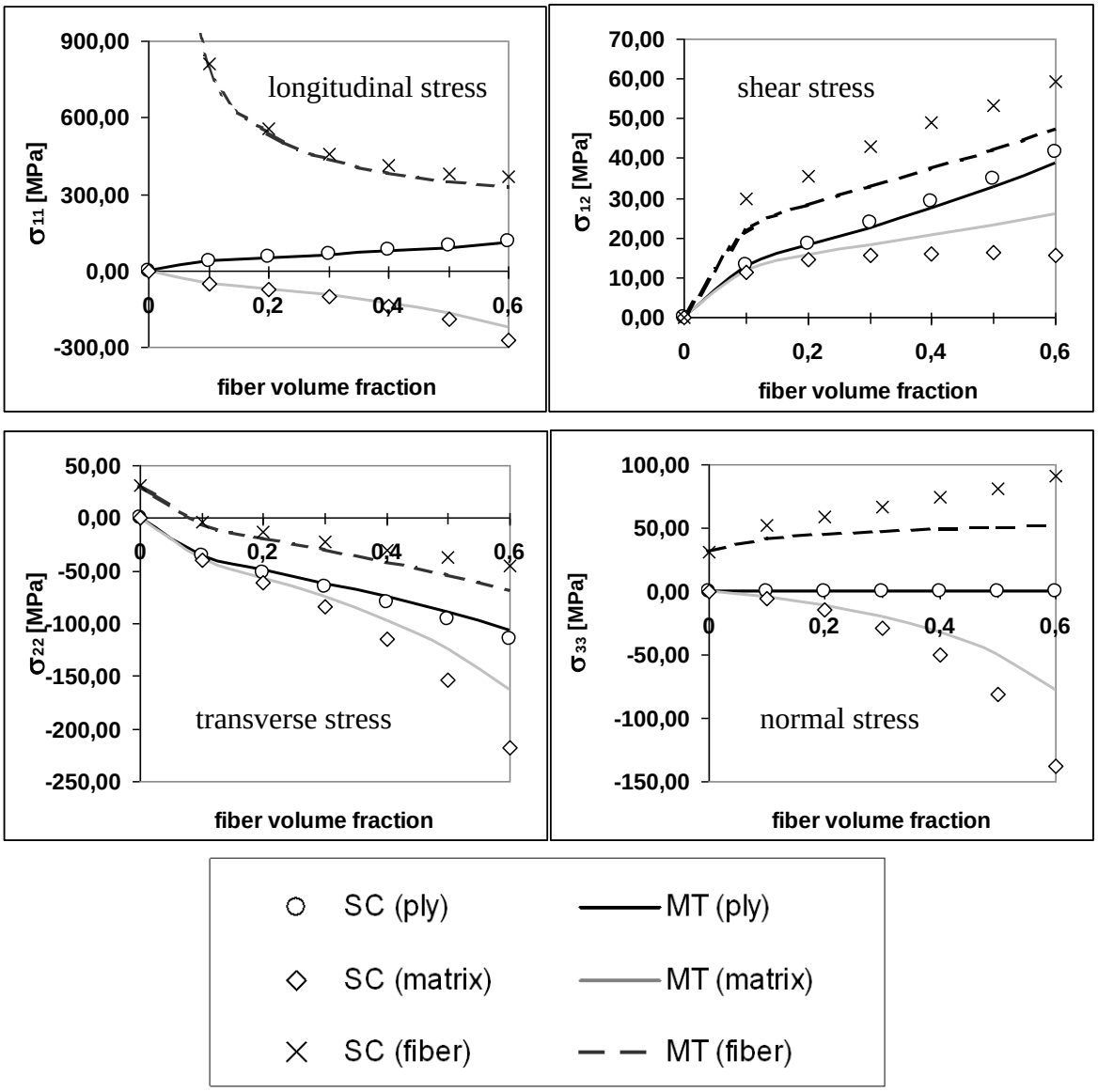

Figure 2. Macroscopic and local stresses in T300/5208 composites for the central ply of $[+55°/-55°]_S$ symmetric laminate, as a function of carbon fiber volume fraction. Comparison between Eshelby-Kröner and Mori-Tanaka predictions.

Tables

	E_1[GPa]	E_2, E_3 [GPa]	ν_{12}, ν_{13}	G_{23} [GPa]	G_{12} [GPa]
T300 fibers [15]	230	15	0.2	7	15
N 5208 epoxy matrix [16]	4.5	4.5	0.4	1.6	1.6

Table 1. Mechanical properties of the constituents of the studied composites.

	β_{11}	β_{22}, β_{33}
T300 fibers [15]	0	0
N 5208 epoxy matrix [16]	0.6	0.6

Table 2. CME for the components of T300/5208 composite.